Topics in Discrete Mathematics Part 2: Introduction to Graph Theory

Definitions: Walks, Circuits, Cycles, etc.

Consider a graph G = (V, E). Any v_i we write down is a vertex $v_i \in V$.

Definition 1.

A walk of length k from v_0 to v_k is a sequence of vertices $v_0v_1v_2\cdots v_{k-1}v_k$ such that the adjacent pairs $v_0v_1, v_1v_2, \ldots, v_{k-1}v_k$ are all edges.

A *trail* is a walk with all edges distinct.

A *path* is a walk with all vertices (and hence all edges) distinct.

Definition 2.

A closed walk is a walk $v_0v_1v_2\cdots v_{k-1}v_0$ from a vertex v_0 back to itself.

A *circuit* is a trail from a vertex back to itself. Equivalently, a circuit is a closed walk with all edges distinct.

A *cycle* is a path from a vertex back to itself (so the first and last vertices are not distinct). Equivalently, a cycle is a closed walk with all vertices (and hence all edges) distinct (except the first and last vertices).

Definition 3.

An *Eulerian circuit* is circuit that uses every edge (exactly once).

A *Hamiltonian cycle* is a cycle that visits every vertex (exactly once, except for the first-and-last vertex, which is visited twice).

Matthew Aldridge m.aldridge@bristol.ac.uk **Topics in Discrete Mathematics** Part 2: Introduction to Graph Theory

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