Topics in Discrete Mathematics
Part 2: Introduction to Graph Theory

Linear Algebra Facts

The following facts from linear algebra will be used without proof.

Throughout, $\mathsf{A} = (a_{ij})$ is an $n \times n$ real matrix.

1. If for some $\lambda \in \mathbb{C}$ and some nonzero vector $\mathbf{v}$ we have $\mathsf{A}\mathbf{v} = \lambda\mathbf{v}$, then we say λ is an *eigenvalue* of A, and $\mathbf{v}$ is an associated *eigenvector*.
2. Eigenvalues can be found as the roots of the *characteristic polynomial* $\chi(x) = \det(x\mathsf{I} - \mathsf{A})$.
3. The *multiplicity* of an eigenvalue λ is the largest t such that $(x-\lambda)^t$ divides the characteristic polynomial $\chi(x)$.
4. When counted with multiplicity, an $n \times n$ matrix has n eigenvalues.
5. If A is *symmetric*, in that $a_{ij} = a_{ji}$ for all i and j, or $\mathsf{A}^\top = \mathsf{A}$, then all the eigenvalues of A are real.
6. If λ is an eigenvalue of A, then λ^k is an eigenvalue of A^k.
7. A *permutation matrix* is a 0–1 matrix with exactly one 1 in each row and in each column.
8. If we simulataneously relabel the rows and columns of A in the same way, the result is the matrix $\mathsf{A}' = \mathsf{P}^{-1}\mathsf{A}\mathsf{P}$ for some $n \times n$ permutation matrix P.
9. If $\mathsf{A}' = \mathsf{Q}^{-1}\mathsf{A}\mathsf{Q}$ for some invertible matrix Q, then A' and A have the same eigenvalues.
10. The *trace* of A is the sum of the diagonal elements $\operatorname{Tr} \mathsf{A} = \sum_{i=1}^n a_{ii}$.
11. If $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigenvalues of A counted with multiplicity, then the trace is the sum of the eigenvalues $\operatorname{Tr} \mathsf{A} = \sum_{i=1}^n \lambda_i$.

Matthew Aldridge
`m.aldridge@bristol.ac.uk`

Topics in Discrete Mathematics
Part 2: Introduction to Graph Theory

Linear Algebra Facts

The following facts from linear algebra will be used without proof.

Throughout, $\mathsf{A} = (a_{ij})$ is an $n \times n$ real matrix.

1. If for some $\lambda \in \mathbb{C}$ and some nonzero vector $\mathbf{v}$ we have $\mathsf{A}\mathbf{v} = \lambda\mathbf{v}$, then we say λ is an *eigenvalue* of A, and $\mathbf{v}$ is an associated *eigenvector*.
2. Eigenvalues can be found as the roots of the *characteristic polynomial* $\chi(x) = \det(x\mathsf{I} - \mathsf{A})$.
3. The *multiplicity* of an eigenvalue λ is the largest t such that $(x-\lambda)^t$ divides the characteristic polynomial $\chi(x)$.
4. When counted with multiplicity, an $n \times n$ matrix has n eigenvalues.
5. If A is *symmetric*, in that $a_{ij} = a_{ji}$ for all i and j, or $\mathsf{A}^\top = \mathsf{A}$, then all the eigenvalues of A are real.
6. If λ is an eigenvalue of A, then λ^k is an eigenvalue of A^k.
7. A *permutation matrix* is a 0–1 matrix with exactly one 1 in each row and in each column.
8. If we simulataneously relabel the rows and columns of A in the same way, the result is the matrix $\mathsf{A}' = \mathsf{P}^{-1}\mathsf{A}\mathsf{P}$ for some $n \times n$ permutation matrix P.
9. If $\mathsf{A}' = \mathsf{Q}^{-1}\mathsf{A}\mathsf{Q}$ for some invertible matrix Q, then A' and A have the same eigenvalues.
10. The *trace* of A is the sum of the diagonal elements $\operatorname{Tr} \mathsf{A} = \sum_{i=1}^n a_{ii}$.
11. If $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigenvalues of A counted with multiplicity, then the trace is the sum of the eigenvalues $\operatorname{Tr} \mathsf{A} = \sum_{i=1}^n \lambda_i$.

Matthew Aldridge
`m.aldridge@bristol.ac.uk`